

Exercise 16

For the following exercises, determine whether there is a minimum or maximum value to each quadratic function. Find the value and the axis of symmetry.

$$f(x) = -x^2 + 4x + 3$$

Solution

Begin by factoring the coefficient of x^2 .

$$f(x) = -(x^2 - 4x - 3)$$

In order to write this quadratic function in vertex form, it's necessary to complete the square, which makes use of the following algebraic identity.

$$(x + B)^2 = x^2 + 2xB + B^2$$

Notice that $2B = -4$, which means $B = -2$ and $B^2 = 4$. Add and subtract 4 on the right side within the parentheses and use the identity so that x appears in only one place.

$$\begin{aligned} f(x) &= -[(x^2 - 4x + 4) - 3 - 4] \\ &= -[(x + (-2))^2 - 7] \\ &= -(x - 2)^2 + 7 \end{aligned}$$

Therefore, the vertex of the parabola is $(2, 7)$. The axis of symmetry is $x = 2$, and the maximum (because the coefficient of x^2 is negative) is $y = 7$.

